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Analysis and Simulation of Two-Phase Fluid Flow in a Porous Medium

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ABSTRACT
A slight improvement in the existing porous media flow equations is of great economic value especially to practitioners in oil exploration and production. We developed two phase immiscible porous media flow equations, taking into account the relevant petrophysical parameters. The Implicit Pressure Explicit Saturation (IMPES) solution approach was employed as a numerical technique to analysed two phase flow equations with application to quarter five spot water flooding scenario, which we simulate with MATLAB.Our model results conforms with what is obtainable with practical scenarios.
Keywords: Two-phase flow, saturation, capillary pressure, Porous media

I. INTRODUCTION

Flow and transport phenomena in porous media is prevalent in the fields of science and engineering with applications in industrial, environmental and biological systems such as (i) the movement of contaminants in the subsurface and their remediation Abriola (1988) (ii) geologic nuclear waste disposal Doughty and Press (1988) (iii) medical application such as brain and lever cancer treatment Ranadhir and Daniel (2013) and most notably (iv) oil recovery from petroleum reservoirs Mohamed and Pramod (2015). These varied applications cover a vast range of length scales; from the kilometer scale in oil and gas recovery to the micron scale for micro-fluidic devices also, temporal scale range from ten thousand years in risk-analysis for long-term isolation of radioactive waste (U.S Environmental Protection Agency 1982) through year-by-year, seasonal, monthly, weekly, daily, and hourly scales for field systems to minutes and even seconds in certain laboratory experiments Rosswall et.al (1988). Two-phase and multiphase flow model in porous media have been developed several decades ago by extending the homogeneous single phase flow through isotropic porous medium postulated by Henry Darcy's in 1856. Several attempts are being made to analyse more and more complex systems by the extension of Darcy's law such as modelling of multiphase flows in petroleum reservoirs Arezou et al. (2019). The heterogeneity of the subsurface of a petroleum reservoir posses a great challenge in the understanding and analyses subsurface flows (Knut-Andreas, 2015; Pan and Miller 2003; Nagi, 2009, (Komal et.al, 2023; Vincent et.al 2022). In order to understand the dynamics of porous media transport , we must have sufficient knowledge of the constitutive relationships between the macroscopic properties of the system such as relative permeabilities, capillary pressures and fluid saturations which are essential in the modeling of the flow transport (Mohammed and Pramod, 2015). The determination of these constitutive relationships are however not without challenge as they are dependent on the fluid properties, the pore space as well as the saturation history. The inherent complexity of pore-scale displacement through the irregular geometry of natural porous media makes the prediction of multiphase flow mechanism in geological processes a very difficult task. Therefore, any scientific approach to this problem would not only require a detailed understanding of the multiphase displacement mechanisms at the micro scale level but must also understand the structure of the porous medium (Pereira et al. 1996). The complexity in the understanding of the pore scale displacement mechanism in the petroleum reservoir, has resulted to a decline in the production of conventional petroleum products, thereby mounting pressure on the discoveries of new oil wells as well as oil exploration in vulnerable areas such as the arctic regions. In the petroleum industry, the economic value of a reservoir is determined by the amount of oil which can be produced from the reservoir, which is affected by either field-scale fluid flow behavior within the porous media as well as pore-scale behaviour of the flow. The pore-scale behaviour of the flow dictates the macroscopic (core-scale) properties of porous media, such as capillary

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pressure as well as the relative permeability. The complicated nature of subsurface flows and transport processes, multiphase flow and heat transfer studies is still poorly understood and analytically intractable (Starikovicius, 2003).

The analysis and understanding of multiphase flow is essential if processes involving multiphase flows are to be optimally and safely designed and controlled. Even small improvements in oil recovery rates can lead to huge economic benefits for the owners of petroleum asset and for this reason much research and engineering activities are designed to improve the understanding of mobilization and displacement mechanisms and to design improved methods for primary and enhanced oil recovery. Mathematical analyses and numerical reservoir simulation play key roles in this endeavor. This informs the motivation behind the present study.

II. MATHEMATICAL FORMULATION:TWO -PHASE IMMISCIBLE FLOW EQUATION

In any petroleum reservoir, there exists at least two different fluid phases. The single phase scenario seldom occurs. Here, we developeda model for the displacement of oil by water. The challenge is that this happens in a simultaneous flow and not with a sharp edge. To circumvent this difficulty, we assumed that there is no mass transfer between the two fluids. We considered two-phase flow where the fluids are immiscible and one fluid phase is considered a wetting phase (the phase which wets the porous medium more) while the other is considered non-wetting. In a water – oil system, water is considered the wetting phase while oil is regarded as the non-wetting phase but in an oil – gas system, oil is considered the wetting phase while gas is the non-wetting phase. We refer to the wetting phase by the subscript w and to the non-wetting phase by the subscript n. In our recent article, Zuonaki and Adokiye (2023) we provided a detailed development of single phase, two phase and three phase flow equations. The mathematical model describing the flow of two phase immiscible fluids in porous media was presented as

$$\nabla \cdot \left[\frac{\rho_n K k_{rn}}{\mu_n} (\nabla p_n - \rho_n G) \right] + q_n = \frac{\partial (\phi \rho_n s_n)}{\partial t} \quad (1)$$

$$\nabla \cdot \left[\frac{\rho_w K k_{rw}}{\mu_w} (\nabla p_w - \rho_w G) \right] + q_w = \frac{\partial (\phi \rho_w s_w)}{\partial t} \quad (2)$$

where ϕ is the rock porosity, ρ_n and ρ_w are the densities of the nonwetting and wetting phase, P_n and P_w are the respective phase pressures, μ_n, μ_w, represent phase viscosities and G represents gravitational term. Also, q_n and q_w are the external mass flow rates for nonwetting and wetting phase respectively while s_n and s_w are the saturations of the nonwetting and wetting phase respectively subject to the constraint equation

$$s_w + s_n = 1 \quad (3)$$

Also, due to the curvature and surface tensionat the interface between the wetting and nonwetting phase, thepressure in the wetting fluid is less than that in the non-wetting fluid Held and Celia (2001). The pressure difference is given by the capillary pressure which is a function of the saturation and the wetting phase Mohammad and Pramod (2015) and is defined by

$$p_{cnw}(s_w) = p_n - p_w \quad (4)$$

III. IMPES FORMULATION OF TWO PHASE INCOMPRESSIBLE FLOW

For practical applications, equations (1) and (2) is formulated as

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$$-\frac{\nabla\cdot(\rho_n u_n)}{\rho_n}-\frac{\nabla\cdot(\rho_w u_w)}{\rho_w}+Q=\frac{\partial\phi}{\partial t}+\phi s_n c_n\frac{\partial p_n}{\partial t}+\phi s_w c_w\frac{\partial p_w}{\partial t} \quad (5)$$

refer to Zuonaki and Adokiye (2023) for details.
Now expanding the first two terms on the left hand side (LHS) of equation(5) result to:

$$-\nabla\cdot u_n-\frac{u_n\cdot\nabla\rho_n}{\rho_n}-\nabla\cdot u_w-\frac{u_w\cdot\nabla\rho_w}{\rho_w}+Q=\frac{\partial\phi}{\partial t}+\phi s_n c_n\frac{\partial p_n}{\partial t}+\phi s_w c_w\frac{\partial p_w}{\partial t} \quad (6)$$

Combining the phase velocities and equating all terms to Q results to

$$\nabla\cdot\left(u_n+u_w\right)+\frac{\partial\phi}{\partial t}+\phi s_n c_n\frac{\partial p_n}{\partial t}+\phi s_w c_w\frac{\partial p_w}{\partial t}+\frac{u_n\cdot\nabla\rho_n}{\rho_n}+\frac{u_w\cdot\nabla\rho_w}{\rho_w}=Q \quad (7)$$

From Darcy's law applied to the different fluid phases, we have

$$u_n=\frac{Kk_{rn}}{\mu_n}\left(\nabla P_n-\rho_n G\right)=K\lambda_n\left(\nabla P_n-\rho_n G\right)$$
$$u_w=\frac{Kk_{rw}}{\mu_w}\left(\nabla P_w-\rho_w G\right)=K\lambda_w\left(\nabla P_w-\rho_w G\right)$$
$$\frac{\partial\phi}{\partial t}=\frac{\partial\phi}{\partial P}\frac{\partial P}{\partial t}=c_r\phi\frac{\partial P}{\partial t} \quad (8)$$
$$c_n=\frac{1}{\rho_n},\ c_w=\frac{1}{\rho_w}$$

Using equation (8) in equation (7)
We have

$$\nabla\cdot\left[K\lambda_n\left(\nabla P_n-\rho_n G\right)+K\lambda_w\left(\nabla P_w-\rho_w G\right)\right]+c_r\phi\frac{\partial P}{\partial t}$$
$$+c_n\left[\nabla P_n\cdot K\lambda_n\left(\nabla P_n-\rho_n G\right)+\phi s_n\frac{\partial P}{\partial t}\right] \quad (9)$$
$$+c_n\left[\nabla P_n\cdot K\lambda_n\left(\nabla P_n-\rho_n G\right)+\phi s_n\frac{\partial P}{\partial t}\right]=Q$$

In this example, we assume that the rock and the two fluid phases are incompressible (that is $c_r=c_n=c_w=0$) and as a result, equation (9) reduces to

$$\nabla\cdot\left[K\lambda_n\left(\nabla P_n-\rho_n G\right)+K\lambda_w\left(\nabla P_w-\rho_w G\right)\right]=Q \quad (10)$$

In equation (10), there are two unknown phase pressures, P_n and P_w. To eliminate one of them, we introduced the capillary pressure defined as $P_{cnw}=P_n-P_w$, which is assumed to be a function of water saturation s_w. Unfortunately, this leads to a rather strong coupling between the pressure equation and the saturation equation. Therefore in other to proceed further, we derive the saturation equation.

3.1 The Saturation Equation

Toderive a complete model, we must derive the equations for the phase saturations s_w and s_n using the continuity equations of each phase. Thus the mass accumulation in a differential volume per unit time is given as

$$\frac{\partial(\phi\rho_\alpha s_\alpha)}{\partial t}$$

Now, assumption that there is no mass transfer between the interphase of the fluid, mass is conserved within each phase. Thus we obtain:

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$$-\nabla\cdot(\rho_n \mathbf{u}_n)+q_n=\frac{\partial(\phi\rho_n s_n)}{\partial t} \qquad (11)$$

$$-\nabla\cdot(\rho_w \mathbf{u}_w)+q_w=\frac{\partial(\phi\rho_w s_w)}{\partial t} \qquad (12)$$

for the non-wetting phase and wetting phase respectively. Again, applying Darcy's law to equations (11) and (12) results to

$$\nabla\cdot\left[\frac{\rho_n K k_{rn}}{\mu_n}(\nabla p_n-\rho_n G)\right]+q_n=\frac{\partial(\phi\rho_n s_n)}{\partial t} \qquad (13)$$

$$\nabla\cdot\left[\frac{\rho_w K k_{rw}}{\mu_w}(\nabla p_w-\rho_w G)\right]+q_w=\frac{\partial(\phi\rho_w s_w)}{\partial t} \qquad (14)$$

Equations (13) and (14) represent the mathematical model describing the flow of two phase immiscible fluids in porous media.
From equation (11) and equation (12) and noting that the flow is incompressible, the saturation equation becomes

$$\phi\frac{\partial s_\alpha}{\partial t}+\nabla\cdot u_\alpha=\frac{q_\alpha}{\rho_\alpha} \qquad \alpha=n,w \qquad (15)$$

However, by equation (3) we need only one saturation equation and it is common practice to choose s_w as the second primary unknown Grader and O'Meara (1988), Arezou et al (2019).

Thus the saturation to be considered is

$$\phi\frac{\partial s_w}{\partial t}+\nabla\cdot u_w=\frac{q_w}{\rho_w} \qquad (16)$$

To connect the continuity equation for water to the pressure equation (10), we need to use what is called the total velocity formulation. To the end, we defind fractional flow of water f_w given as:

$$f_w=\frac{u_w}{u_t} \text{ or } u_w=f_w(s_w)u_t \qquad (17)$$

Substituting equation (17) into equation (16) yields

$$\phi\frac{\partial s_w}{\partial t}+\nabla\cdot f_w(s_w)u_t=\frac{q_w}{\rho_w} \qquad (18)$$

3.2Model Example

The aim of this illustration is to apply equations (10) and (18) to stimulate water injection through an injection well of an oil filled reservoir thereby displacing the oil towards the production well. We apply the quarter five-spot problem which is a standard test case for numerical methods in reservoir simulations.

Figure 1: Quarter five-spot formulation

We assumed the permeability to be homogeneous and isotropic with K ≡ 1 for all $x\in\mathbb{R}^2$. We place an injection well at the origin and production wells at the points (±1, ±1) and specify no-flow conditions at the boundaries. The boundary conditions ensure that the flow is identical and as if werepeated the five-spot well pattern to infinity in every direction. The flow inthe five-spot is symmetric about both the coordinate axes. Thus we reduce the computational domain to a quarter, which corresponds to the unit box $\Omega=[0,1]^2$. We disregard gravity and capillary forces. Under these assumptions, equations (10) and (18) reduce to the following:

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$$\nabla \cdot K\lambda(s_w)\nabla P = Q \qquad (19)$$

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot f_w(s_w)u_t = \frac{q_w}{\rho_w} \qquad (20)$$

3.3Numerical Scheme

Let the total time interval $[0,\mathrm{T}]$ be divided in N time steps as $0 < t^0 < t^1 < \ldots < t^N = T$. Define the time step length $\Delta t^i = t^{i+1} - t^i$. The numerical scheme for equations (19) and (20) are given as follows:

$$\nabla \cdot K\lambda(s_w^i)\nabla P^{i+1} = Q^{i+1} \qquad (21)$$

$$\phi \frac{\left(s_w^{i+1} - s_w^i\right)}{\Delta t^i} + \nabla \cdot \left(f_w^{i+1} u_t^{i+1} \right) = \frac{q_w^{i+1}}{\rho_w} \qquad (22)$$

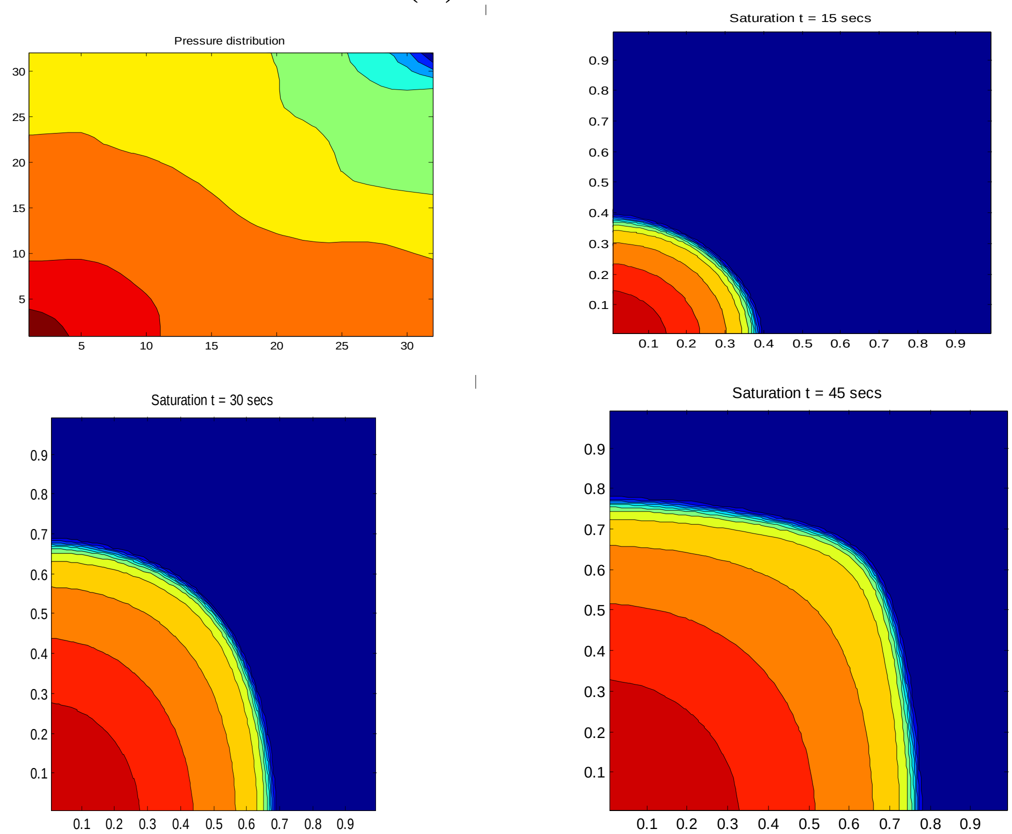
Finally, we introduce the following saturation dependent quantities defined as:

$$\lambda_w(s) = \frac{(s^*)}{\mu_w}, \quad \lambda_o(s) = \frac{(1 - s^*)}{\mu_o}, \quad s^* = \frac{(s - s_{wc})}{1 - s_{or} - s_{wc}} \qquad (23)$$

Here s_{or} is the irreducible oil saturation, that is, the lowest oil saturation that can be achieved by displacing oil by water, and s_{wc} is the connate water saturation, that is, the saturation of water trapped in the pores of the rock during formation of the rock. We develop a MATLAB code to simulate equations (21) and (22).

IV. SIMULATION OF HYPOTHETICAL PROBLEM WITH MATLAB

We considered a reservoir with dimension $(64 \times 64 \times 1)$ ft; initially filled with oil. To produce the oil in the upper-right corner, we inject water in the lower left. For simplicity, we assumed unit porosity, unit viscosities for both phases, and set $s_{or} = s_{wc} = 0$. In this non-dimensional model, it takes unit time to inject one pore-volume of water, i.e., the unit time corresponds to the number of injected pore volumes of water. The simulated results of the pressure distribution and the saturation evolutions during the displacement process are shown in figure 2.



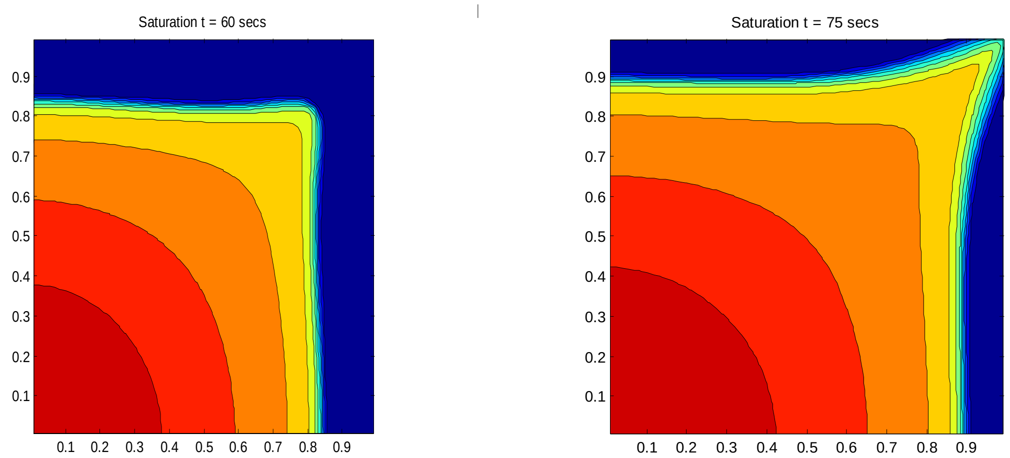


Figure 2: Pressure and saturation profiles for the simulation of our hypothetical problem

The algorithm is quite simple. First we set up the grid, the fluid properties and generate the initial saturation distribution. Thereafter, the solution is advanced with time by repeating the following two steps: (i) solve the pressure equation and compute the edge velocities and (ii) using the fixed edge velocities, solve the fluid transport equation at a time step Δt.

In figure 2, we have plotted the initial pressure and saturation profiles at five equally-spaced time levels (that is, 15 seconds per interval) as computed on a uniform $64\times64\times1$ grid. The pressure distribution figure clearly shows that the pressure reduces as it advances from the injection well at the lower left hand side towards the upper right hand side. This is what is expected in practical applications. The saturation profile consists of a leading shock-wave, in which water immediately displaces a fraction of the oil in the reservoir. Behind the front, the water saturation is increasing monotonically, meaning that more oil is gradually displaced as more water is injected. Close to the injector, the level curves are almost circular (graph of saturation at t = 15 seconds), corresponding to the circular symmetry in pressure at the injector. As more water is injected, the leading water front develops a finger extending toward and finally breaking through to the production well at saturation t = 75 seconds.

V. CONCLUSION

In this research, we analysed two phase flow equations in a porous medium. The mass balance equation for each fluid phase, darcy's law was modified to accommodate the different fluid phases. Our flow equations were transform into pressure and saturation formulations and by rigorous mathematical applications, we are able to develop reservoir flow equations for two phase immiscible flows. Also, we discretized and simulate two phase flow equations and investigate the pressure distribution and saturation evolutions. Our results are in line with what is obtainable in practical scenarios.

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